Problem set is due in class on Oct. 12th. You must show your work to receive credit.

1. Suppose X is a random sample from $U(\theta_1, \theta_2)$, where both parameters are unknown ($-\infty < \theta_1 < \theta_2 < \infty$). Find the two MLEs.

2. Suppose that a random variable has an exponential distribution with mean θ, which is unknown ($\theta > 0$). Find the Fisher information $I(\theta)$.

3. Suppose that a single observation X is taken from

$$U\left[\theta - \frac{1}{2}, \theta + \frac{1}{2}\right].$$

   Let the rival hypotheses be $H_0 : \theta \leq 3$ and $H_1 : \theta \geq 4$. Construct a test for which the power function is 0 for $\theta \leq 3$ and 1 for $\theta \geq 4$.

4. Suppose that $X_1, \ldots, X_n$ are a random sample from $N(\mu, 4)$. Consider the two hypotheses $H_0 : \mu = -1$ versus $H_1 : \mu = 1$. Find the minimum value of $\alpha + \beta$ that can be attained for each of the following values of the sample size n: 1, 4, 16, 36.

5. A random sample of size 20 is taken from a normal distribution with unknown mean and a known variance, $\sigma^2 = 5$.

   (a) Find the UMP test at $\alpha = 0.5$ for $H_0 : \mu = 7$ v. $H_1 : \mu > 7$.

   (b) Find the power of the test at alternatives of 7.5, 8, 8.5, and 9.

   (c) Graph the power function.

6. Suppose 9 randomly selected observations are drawn from $N(\mu, \sigma^2)$. The sample mean is 22 and the sum of squared deviations is 72. Let $\alpha = 0.05$ and carry out the following tests.

   (a) $H_0 : \mu \leq 20$ v. $H_1 : \mu > 20$

   (b) $H_0 : \mu = 20$ v. $H_1 : \mu \neq 20$

   (c) Construct the 95% confidence interval.