Problem set is due on Sept. 14th before class. You must show your work to receive credit.

1. Let $F(x) = e^{x-3}$ for $x \leq 3$ and 1 otherwise. Find and sketch the p.d.f. of X.

2. Let X have a uniform distribution on $[a, b]$. Determine the moment generating function of X.

3. Let X and Y have the joint distribution

$$f(x, y) = \frac{1}{30}(x + y)$$

for $x = 0, 1, 2$ and $y = 0, 1, 2, 3$.

   (a) Determine the marginals.

   (b) Are X and Y independent?

4. Let a joint density be

$$f(x, y) = \frac{1}{3}(x + y), \quad \text{for } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 2$$

What is the variance of $2X - 3Y + 8$?

5. Suppose that a random variable X can have each of seven values -3, -2, -1, 0, 1, 2, 3 with equal probability. Determine the density of $Y = X^2 - X$.

6. Suppose the pdf of a random variable X is

$$f(x) = \begin{cases} 3x^2 & 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find the pdf of $Y = 1 - X^2$.

7. Suppose that X_1 and X_2 are independent random variables each distributed $U(0, 1)$. Find the pdf of $Y = X_1 + X_2$.