Problem set is due Tuesday, Jan. 29th in class.

1. Write down the following matrices:

$$\begin{aligned}\mathbf{A} &= \{a_{ij}\} \text{ for } a_{ij} = i + j \text{ for } i = 1, 2, 3 \text{ and } j = 1, 2\\ \mathbf{B} &= \{b_{kt}\} \text{ for } b_{kt} = k^{t-1} \text{ for } k = 1, \ldots, 4 \text{ and } t = 1, \ldots, 3\\ \mathbf{C} &= \{c_{rs}\} \text{ for } c_{rs} = 3r + 2(s-1) \text{ for } r = 1, \ldots, 4\end{aligned}$$

2. Expand the matrix product

$$\mathbf{X} = \{[\mathbf{AB} + (\mathbf{CD})'][(\mathbf{EF})^{-1} + \mathbf{GH}]\}'.$$

   Assume that all matrices are square and that $\mathbf{E}$ and $\mathbf{F}$ are nonsingular.

3. Calculate $|\mathbf{A}|$, $\text{tr}(\mathbf{A})$, and $\mathbf{A}^{-1}$ for

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & 7 \\ 3 & 2 & 5 \\ 5 & 2 & 8 \end{bmatrix}$$

4. What operation is performed by postmultiplying a matrix by a diagonal matrix? What about premultiplication?

5. Are the following quadratic forms positive for all values of $\mathbf{x}$?

   (a) $y = x_1^2 - 28x_1x_2 + (11x_2)^2$

   (b) $y = 5x_1^2 + x_2^2 + 7x_3^2 + 4x_1x_2 + 6x_1x_3 + 8x_2x_3$

6. Explain why $\mathbf{X}'\mathbf{X}\mathbf{G}\mathbf{X}'\mathbf{X} = \mathbf{X}'\mathbf{X}$ implies that $\mathbf{X}\mathbf{G}\mathbf{X}'\mathbf{X} = \mathbf{X}$.

7. For a square matrix $\mathbf{A}$, suppose there is an $\mathbf{x} \neq \mathbf{0}$ s.t. $\mathbf{Ax} = \mathbf{0}$. Explain why $\mathbf{A}$ is singular.

8. Download the German Weimar Republic Data, 1920-1933 from the ICPSR web site and demonstrate that you have read it into R. Success requires a number of steps. Go to the "Data & Documentation" tab on the web site, and download the "ASCII+SPSS Setup" zip file. You will then need to go to the Star Lab and use SPSS. Instructions are here. Export the data in your preferred format and read it into R.